General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2B

MM2B

Thursday 16 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

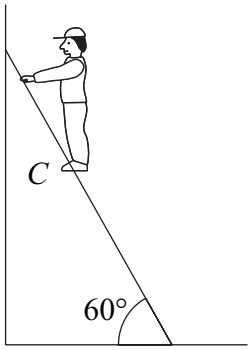
1 A light spring of natural length 0.4 metres and modulus of elasticity λ newtons lies on a smooth horizontal table. One end of the spring is attached to the table at a point O and the other end is attached to a particle. The particle is held in equilibrium on the table by a horizontal force of magnitude 12.5 newtons. In this position the length of the spring is 0.5 metres.

(a) Show that $\lambda = 50$. *(3 marks)*

(b) The horizontal force is removed and the particle, which is of mass 0.2 kg, is released from rest and moves towards O. Calculate its speed when the spring reaches its natural length. *(5 marks)*

2 A uniform ladder of length 3 metres and mass 35 kg rests in equilibrium with its foot on a rough horizontal floor and its top leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the angle between the ladder and the floor is 60°.

A man of mass 90 kg is standing at a point C on the ladder. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the floor is 0.3. The man may be modelled as a particle at C.

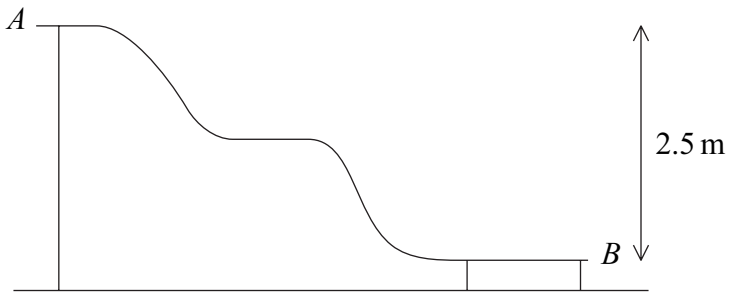


(a) Draw a diagram to show the forces acting on the ladder. *(1 mark)*

(b) Find the magnitude of the frictional force between the ladder and the ground. *(3 marks)*

(c) Show that the man is just over half way up the ladder. *(6 marks)*

3 The diagram below shows the cross-section of a slide in a children's playground. Jan, of mass 28 kg, sits at rest on the slide at *A*.



Jan's friend gives her a push which causes her to start moving with speed $1\,\text{m}\,\text{s}^{-1}$. She then slides down the slide to *B*, which is 2.5 metres below the level of *A*.

(a) In a simple model, Jan's motion is not affected by any forces of resistance.

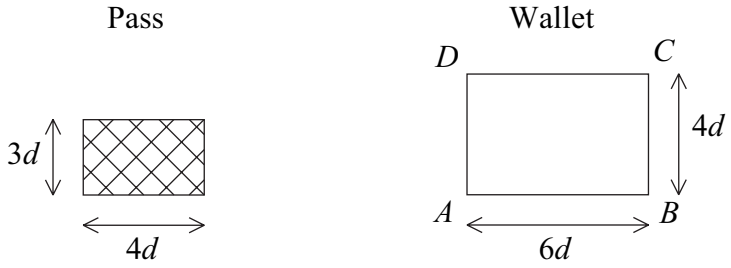
(i) By using energy, show that Jan's speed when she reaches *B* is $7.07\,\text{m}\,\text{s}^{-1}$, correct to three significant figures. *(4 marks)*

(ii) It takes *T* seconds for Jan to move from *A* to *B*. Sketch a velocity-time graph for Jan's motion as she travels from *A* to *B*. *(3 marks)*

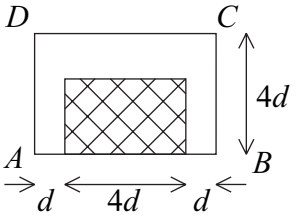
(b) In a more refined model, Jan's motion is assumed to be affected by resistance forces. Given that the work done by the resistance forces while Jan travels from *A* to *B* is 350 J, calculate Jan's speed when she reaches *B*. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

4 A security pass, of length $4d$ and width $3d$, consists of a rectangular piece of plastic. The pass can be carried in a transparent plastic wallet, $ABCD$, where $AB = 6d$ and $BC = 4d$. The pass and the wallet, which may be modelled as uniform laminas, are shown in the diagrams.



The pass is placed inside the wallet with its longer edge along AB and its centre equidistant from BC and AD, as shown in the diagram below.



The mass of the pass is $4M$ and the mass of the wallet is M.

(a) Show that the distance of the centre of mass of the pass and wallet above AB is $1.6d$. *(4 marks)*

(b) The wallet, with the pass still inside and in the position shown in the diagram, is suspended freely from the point C. Find the angle, in degrees, that the edge CD makes with the vertical. *(4 marks)*

5 A particle travels along a straight line. At time t, the velocity of the particle is v. When $t = 0$, $v = u$. The acceleration of the particle throughout the motion is $\frac{k}{v}$, where k is a constant.

Show that

$$v^2 = u^2 + 2kt$$ *(6 marks)*

6 A motorcyclist rides her motorcycle at a constant speed on a horizontal circular path around a roundabout.

(a) The radius of the circular path of the motorcycle is 15 metres.

(i) Calculate the magnitude of the acceleration of the motorcycle when it is moving at $7.5\,\mathrm{m\,s^{-1}}$. *(2 marks)*

(ii) The total mass of the motorcycle and rider is 400 kg. To avoid skidding taking place, the magnitude of the frictional force between the motorcycle and the road must not exceed 2940 N. Find the greatest speed, $V\,\mathrm{m\,s^{-1}}$, at which the motorcyclist can ride her motorcycle around this roundabout without skidding. *(3 marks)*

(b) Without referring to air resistance, state one modelling assumption you have made in answering part (a). Explain why your assumption is reasonable. *(2 marks)*

(c) In wet weather the maximum frictional force between the motorcycle and the road is reduced. The motorcyclist wishes to ride her motorcycle at the speed $V\,\mathrm{m\,s^{-1}}$, as found in part (a)(ii), around the roundabout. State whether she should increase or decrease the radius of her circular path to avoid skidding, giving a reason for your answer. *(2 marks)*

7 A particle P moves in a horizontal plane so that at time t seconds after the motion begins its position vector $\mathbf{r}$ metres is given by

$$\mathbf{r} = \sin 2t\,\mathbf{i} + 6t\,\mathbf{j}$$

(a) (i) Find the velocity of P at time t. *(2 marks)*

(ii) Show that the speed of P at time t is

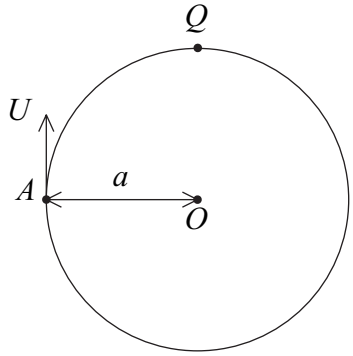
$$\sqrt{(4\cos^2 2t + 36)}$$ *(3 marks)*

(iii) Find the first value of t for which the speed of P has its minimum value. *(2 marks)*

(b) (i) The mass of P is 0.25 kg. Find the force, $\mathbf{F}$ newtons, acting on P at time t. *(3 marks)*

(ii) State the direction of $\mathbf{F}$ and explain why its magnitude never exceeds one newton. *(2 marks)*

8 A smooth circular wire, of centre O and radius a, is fixed in a vertical plane. A small bead, of mass m, is free to move on the wire. The bead is projected upwards, with initial speed U, from a point A on the wire which is level with O, as shown in the diagram.



(a) Find, in terms of a and g, the least value of U required for the bead to reach the point Q, at the top of the circular wire, in the subsequent motion. *(3 marks)*

(b) If $U = \sqrt{\frac{5ag}{2}}$, find the vertical height, h, above the level of O at which the magnitude of the normal reaction between the bead and the wire is zero. *(8 marks)*

END OF QUESTIONS

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